

THE USE OF THE MATHEMATICAL CURVES IN THE OBTAINING PROCESS OF THE SPECIFIC ORNAMENTS OF THE DECORATIVE ELEMENTS FROM A GARDEN

UTILIZAREA CURBELOR MATEMATICE ÎN PROCESUL DE OBTINERE A ORNAMENTELOR SPECIFICE ELEMENTELOR DECORATIVE DINTR-O GRĂDINĂ

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Abstract. *The decorative and non-decorative elements from a garden can have different ornaments. For getting these it may be used, various mathematical curves. Starting from this observation, the authors analyzed some mathematical curves from the perspective of used them within the Inventor computer program, with the goal of generated different ornaments using this software. Following this study, they arrived at some conclusions and recommendations, presented at the end of this paper, in the case of use mathematical curves by the Inventor software.*

Key words: ornaments, mathematical curves, Inventor

Rezumat. *Elementele decorative sau non-decorative dintr-o grădină pot avea diferite ornamente. Pentru obținerea acestora pot fi utilizate diferite curbe matematice. Pornind de la această observație, autorii au analizat mai multe curbe matematice din perspectiva utilizării lor în cadrul programului de calcul Inventor, cu scopul de a genera diferite ornamente. În urma acestui studiu ei au ajuns la câteva concluzii și recomandări utile, prezentate la sfârșitul acestui articol, în cazul utilizării curbelor matematice de către programul de calcul Inventor.*

Cuvinte cheie: ornamente, curbe matematice, Inventor

INTRODUCTION

For the design of the decorative and non-decorative elements from a garden, the Autodesk Inventor software can be successfully used (Prună and Slonovski, 2018). These may have, or not, ornaments inside of them. In this context, it is very important to know what is the level of precision that can be gained when comes about to be designed the ornaments within the decorative elements desired. The authors tried to find out if it is possible to be obtained a complex artistic model in case that the designer has in his mind such a thing.

It is known the fact that the shape of an ornament may be defined starting from one or more curves. So, the authors concluded that the way of representing different curves, in Inventor software, is the answer to the question mentioned above. In this software, some curves can be obtained using a command called

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Spline Interpolation. To use this command, the positions of some points must be precisely known. These points are on the curve. In other words, the points belonging to the curve. So, the user must specify the "x" and "y" coordinates of each point used in the process of representing the desired curve. In some cases, because of the large number of dimensions that must be given, this task can be difficult to accomplish. This difficulty lies in the following two causes: the first is related to the accuracy with which the coordinates of a point can be determined and the second is related to the specification in the software of a large number of dimensions.

Thus, when the designer imagines a particular shape for an ornament, he must to imagine and draw on a sheet of paper, the curve that defines it. Then he has to attach to that curve an orthogonal system of axes. Finally, he must accurately determine the coordinates of some points belonging to the drafted curve. The more points there are, the better the accuracy. But, we must not lose sight of the fact that the operation by which the coordinates of the points are measured depends both on the accuracy of the tools used and on the skills of the designer.

Regarding the second case, it must be shown that it is difficult to dimension a large number of points. This is due to the space available in the graphics editor of the software, limited by the surface of the monitor. This fact leads to the overlapping of the dimensions and, by consequence, the correctness of them will be hard to track, figure 1 and figure 2.



Fig. 1 A shape obtained started from two free curves.

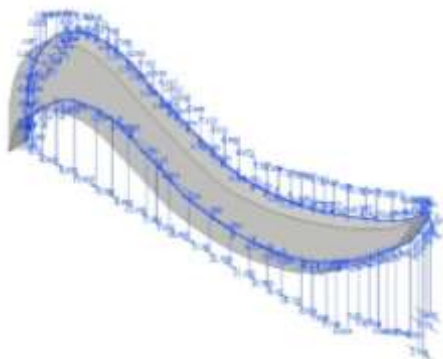


Fig. 2 The dimensions used to obtain the two free curves.

At this point, the authors wondered to what extent different mathematical curves can be used successfully, thus having the opportunity to eliminate the shortcomings listed above. So, they decided to study different mathematical curves, from the perspective of using them in the process of the ornaments' generation.

MATERIAL AND METHOD

A curve can be obtained with the command named Equation curve. Having in mind the idea that a curve it must also have an aesthetic role, the authors chose the

next mathematical curves.

The **Maltese Cross** is a well-known symbol, and cartesian parametrization for the curve is shown below (<https://mathcurve.com/courbes2d.gb/croixdemalte/croixdemalte.shtml>):

$$x = a * \cos(t)(\cos^2(t) - 2) \text{ and } y = a * \sin(t) * \cos(t) \quad (1)$$

This curve could be obtained in at least two ways shown in figure 3 and Figure 4. If the option presented in figure 3 is chosen, it must be immediately used the Mirror command but, finally, the extrusion operation will be not possible. So, the user has to choose the option shown in figure 4.



Fig. 3 This approach, after completing the curve, will not allow the possibility to use the Extrude command.



Fig. 4 The solution will give the possibility to use the Extrude command.

The cartesian parametrization for **Parabola** is shown below (<http://www.math.md/stireal/matematica/candidat/parabola.pdf>):

$$x = \frac{t^2}{a} \text{ and } y = t(2)$$

A parabola like that in figure 5 could be obtained using equations(2) for an interval $-t$ to t where t is a real number. This curve must be closed, and two constraints must be established to make possible the use of the Extrude command, figure 6.



Fig. 5 The parabola

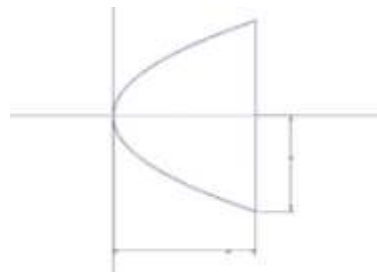


Fig. 6 The curve o parabola must be closed

The cartesian parametrization for **Hyperbola** is shown below (<https://mathcurve.com/courbes2d.gb/hyperbole/hyperbole.shtml>):

$$x = 3/\cos(t) \text{ and } y = 4 * (\sin(t)/\cos(t))(3)$$

A hyperbola like that in figure 7 could be obtained using equations(4) for an interval $-t$ to t where t is a real number. This curve must be closed, and two constraints

must be established to make possible the use of the Extrude command, figure 8.



Fig. 7 The hyperbola

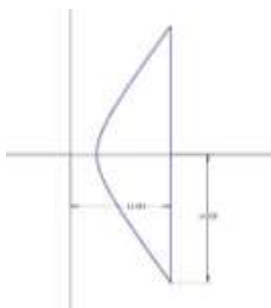


Fig. 8 The curve of hyperbola must be closed

There are several mathematical curves whose cartesian parametrization is known but which, transformed into closed curves, do not allow the extrusion of the surface defined by them. This happens even though the software displays the message "Fully constrain", which indicates that 3D modelling operations can take place. Here are some such curves: **Cissoid of Diocles** with cartesian parametrization

given by equations (4), (<https://mathcurve.com/courbes2d.gb/cissoiddroite/cissoiddroite.shtml>), **Piriform Quartic** (Drop fo Water) with cartesian parametrization given by equations (5), (<https://mathcurve.com/courbes2d.gb/piriforme/piriforme.shtml>) **Deltoid** with cartesian parametrization given by equations (6), (<https://mathcurve.com/courbes2d.gb/deltoid/deltoid.shtml>) and **Astroid** with cartesian parametrization given by equations (7), (<https://mathcurve.com/courbes2d.gb/astroid/astroid.shtml>).

$$x = 2 * a * \sin^2(t) \text{ and } y = (2 * a * \sin^3(t)) / \cos(t) \quad (4)$$

$$x = a * \cos^2(t) \text{ and } y = a^2 * \cos^3(t) * \sin(t) \quad (5)$$

$$x = a * (2 * \cos(t) + \cos(2t)) \text{ and } y = a * (2 * \sin(t) - \sin(2t)) \quad (6)$$

$$x = a * \cos^3(t) \text{ and } y = a * \sin^3(t) \quad (7)$$

The cartesian equations and a range of values were entered, for each studied curve, figure 9. Having these data, the software automatically draws the curve for the specified range, figure 10.

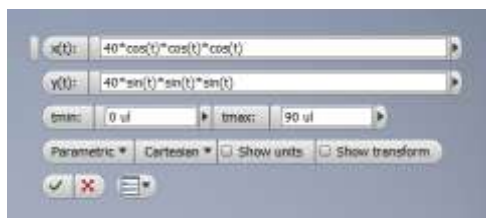


Fig. 9 The data for a branch of an astroid curve

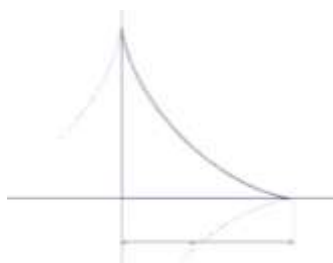


Fig. 10 The branch of an astroid curve

Then, to determine if the curve can be or not extrude, the resulting curve was transformed into a closed curve. This was done in two ways and, each time the result was tested in connection with the possibility of extruding the resulting surface. A first option was to close the obtained curve with the help of the right segments, as can be seen in figure 10. The second option consisted of the use of the Mirror command, the curves being symmetrical about one of the OX or OY axes or with both.

RESULTS AND DISCUSSIONS

Analysing the mathematical curves mentioned above, the authors found that there are curves that can be transformed into closed curves and the resulting surfaces allow the execution of the Extrude command. They are represented with maximum precision by the software and allow the ornaments to be obtained with relative ease.



Fig. 11 The Maltese Cross

The Maltese Cross, as an ornament, can be obtained starting from the curve shown in figure 4. After extrusion, the result can be multiplied by Circular Pattern and the Maltese Cross is obtained, figure 11. A model that is based on the parabola curve, figure 12, was obtained starting from the result presented in figure 6. The model from figure 13 was obtained starting from the result presented in figure 8.



Fig. 12 Parabola use

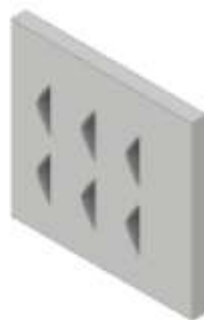


Fig. 13. Hyperbola use

For those curves that do not allow to obtain surfaces that could be extruded, the authors have found a way of working that still allows their use. Thus, the curve to be used as the basis of the model is represented, according to its equations, in the Inventor software. Then several points are chosen on the respective curve and their coordinates are determined. To establish the coordinates, the function that allows the placement of dimensions is used. This ensures a high accuracy of measurements. The results may or may not be retained in an Excel file. A polyline is drawn through these points. Subsequently, it allows the construction of surfaces that can be extruded. If the coordinates of the points have been saved using an Excel file, there is a command that allows plotting the points that belong to the curve by automatically reading the coordinates of those points in that file.

Figure 14 shows a model for the construction of which several cisoid curves were used. The technique mentioned above was used to obtain each curve. Once again, the technique of determining the coordinates of the points

belonging to the desired curve was used. A model for which astroid curves were used is shown in figure 15. Once again, the technique of determining the coordinates of the points belonging to the desired curve was used.



Fig. 14 At the basis of this model, there are four cissoid curves



Fig. 15 At the basis of this model, there are four astroid curves

CONCLUSIONS

1. When the Inventor software is used to build ornaments inside decorative and non-decorative elements from a garden, some mathematical curves can be used. These can be represented with high precision by the software, a fact that guarantees the user that he will obtain the geometric shapes designed by them, with the respect of the finest details.

2. The authors proposed a way of working that allows the use of even those mathematical curves that do not allow that starting from them to generate a volume model, the use of the Extrude command being not possible in this case. This way of working ensures the required accuracy by using other functions specific to this software that allow determining, with high precision, the coordinates of some points that belong to the respective curve.

3. Free curves will be obtained with low accuracy. This is due to the fact that the accuracy with which the coordinates of some points on the respective curve can be measured is low. It depends both on the accuracy of the tools used but also on the skills of the designer.

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